

# Simulation of Dynamic Characteristic of $L$ -branch Selection Combining Diversity Receiver in Nakagami- $m$ Environment

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**Abstract** – In this paper,  $L$ -branch selection combining (SC) diversity receiver, as powerful technique for mitigating an influence of multipath fading and cochannel interference (CCI), is considered in this paper. Average fade duration, as important dynamic characteristic, of this system in Nakagami- $m$  fading environment is simulated using the sum-of-sinusoids-based Nakagami- $m$  simulator. Simulation results show great agreement with earlier published numerical results.

**Keywords** – Fading, Selection combining diversity, Average fade duration, Sum-of-sinusoids-based simulator.

## I. INTRODUCTION

Multipath fading due to multipath propagation and cochannel interference (CCI) as a result of frequency reuse which is essential in increasing cellular radio capacity are the main factors limiting system's performance [1]. Several statistical models are used to describe fading in wireless environments: Rayleigh, Nakagami- $m$ , Rician, and Weibull. Nakagami- $m$  distribution contains a set of other distributions as special cases and provides optimum fits to collected data in indoor and outdoor environments [2]. Moreover, it can model signal in severe, moderate, light and no fading environment via adjusting its parameter  $m$ . Having in mind all of that, there are a huge number of papers considering the performance of wireless systems over Nakagami- $m$  fading channels.

Space diversity techniques, which combine input signals from multiple receive antennas, are the well known techniques that can be used to upgrade transmission reliability and increase channel capacity without increasing transmission power and bandwidth [3]. The most popular space diversity techniques are selection combining (SC), equal-gain combining (EGC), and maximal-ratio combining (MRC) [4]. In opposition to MRC and EGC, SC receiver is simpler for practical realization because it processes only one of the diversity branches. Traditionally, SC receiver selects the branch with the highest signal-to-noise ratio (SNR), or equivalently, with the strongest signal assuming equal noise power among the branches. However, in interference-limited environment, SC receiver can apply one of following decision algorithms: desired signal power algorithm, total signal power algorithm, and signal-to-interference ratio (SIR) algorithm. Desired signal power algorithm for an interference-limited SC system has identical performance as the total signal power algorithm over entire range of average SIR [5]. In addition,

implementation of total power algorithm is the most practical among all decision algorithms, but desired signal algorithm is easier for mathematical modelling.

In this paper, motivated by the previous observations, dynamic characteristic of desired power signal based  $L$ -branch SC diversity receiver operating over Nakagami- $m$  fading environment in the presence of CCI is modeled and simulated using program package Matlab. The Nakagami- $m$  fading simulator incorporating Pop's architecture with Zhang decomposition algorithm is used [6]. In other words, a random phase into low-frequency oscillators for gaining the wide-sense stationary property is inserted, while decomposing a real number of the fading figure,  $m$ , into two parts, an integer and a fraction, is introduced to accomplish design [7]. The average fade duration (AFD) of considered system is simulated to reflect the correlation properties of fading channels and provide a dynamic representation of the system outage performance. Furthermore, simulation results are compared with previously published numerical results in papers [8], [9].

## II. NUMERICAL RESULTS

The instantaneous SIR at the output of SC system applying desired signal power algorithm is given by  $\eta = \max\{r_1^2, r_2^2, \dots, r_L^2\} / a^2 = r^2 / a^2$ , where  $r_i$  is desired signal envelope on  $i$ -th diversity branch and  $a$  is CCI envelope at selected branch.

The AFD corresponds to average length of time in which envelope remains under given value, known as threshold. In interference-limited environment, the respective AFD at threshold  $\mu$ ,  $\mu = \sqrt{\eta}$ , is defined as [10]

$$T_\mu(\mu) = F_\mu(\mu) / N_\mu(\mu), \quad (1)$$

where  $F_\mu(\mu)$  and  $N_\mu(\mu)$  denote the cumulative distribution function (CDF) and average level crossing rate (LCR) of the envelope ratio, respectively. The average LCR of the envelope ratio of desired signal and CCI,  $\mu$ , at threshold  $\mu_{th}$  is defined as the rate at which a fading process crosses level  $\mu_{th}$  in a positive (or negative) going direction and is mathematically defined by the Rice's formula [10]

$$N_{\mu}(\mu_{th}) = \int_0^{\infty} \dot{\mu} p_{\mu\dot{\mu}}(\mu_{th}, \dot{\mu}) d\dot{\mu}, \quad (2)$$

where  $\dot{\mu}$  denotes the time derivative of  $\mu$  and  $p_{\mu\dot{\mu}}(\mu, \dot{\mu})$  is the joint probability density function (PDF) of random variables  $\mu(t)$  and  $\dot{\mu}(t)$  in an arbitrary moment  $t$ .

Expressions for the average LCR of dual and triple SC diversity system applying desired signal power decision algorithm over Nakagami- $m$  fading channels in the presence of CCI are presented in [11], [9] as

$$N_{\mu}(\mu_{th}) = \sum_{k=0}^{\infty} \frac{\sqrt{2\pi} f_m m^{m+k-0.5} m_l^{m_l-0.5} \rho^k \mu_{th}^{2m+2k-1}}{k! \Gamma(m) \Gamma(m_l) (1-\rho)^k} \times \left[ \frac{2\beta \Gamma(m+m_l+k-0.5)}{(\delta+\gamma)^{m+m_l+k-0.5}} - \sum_{n=0}^{m+k-1} \frac{m^n \mu_{th}^{2n} \Gamma(m+m_l+n+k-0.5)}{n! (1-\rho)^n} \times \frac{2\beta}{\Omega_s^n (2\delta+\gamma)^{m+m_l+k-n-0.5}} \right] \quad (3)$$

and

$$N_{\mu}(\mu_{th}) = \frac{\sqrt{2\pi} f_m \mu_{th}^{2m-1} m_l^{m_l-0.5} m^{m-0.5} S^{m_l-0.5}}{\Gamma(m_l)} \times \sqrt{S m_l + m \mu_{th}^2} \sum_{i,j=0}^{\infty} \theta^i \alpha \left[ \frac{2\Gamma(i+j+m)}{\Gamma(j+m)(1+\rho)^{i+j+m}} \times \left( \frac{\Gamma(j+m+m_l-0.5)}{\alpha_1^{j+m+m_l-0.5}} - \sum_{k=0}^{i+m-1} \frac{\Gamma(j+m+m_l+k-0.5) \theta^k}{k! \alpha_2^{j+m+m_l+k-0.5}} - \sum_{l=0}^{j+i+m-1} \frac{\Gamma(j+m+m_l+l-0.5) \theta^l (1+\rho)^l}{l! \alpha_3^{j+m+m_l+l-0.5}} + \sum_{k=0}^{i+m-1} \sum_{l=0}^{j+i+m-1} \frac{\Gamma(j+m+m_l+k+l-0.5) \theta^{k+l} (1+\rho)^l}{k! l! \alpha_4^{j+m+m_l+k+l-0.5}} \right) + \theta^i \left( \frac{\Gamma(i+j+m+m_l-0.5)}{\alpha_5^{i+j+m+m_l-0.5}} - \sum_{l=0}^{j+i+m-1} \frac{\Gamma(i+j+m+m_l+l-0.5) \theta^l}{l! \alpha_3^{i+j+m+m_l+l-0.5}} - \sum_{k=0}^{i+m-1} \frac{\Gamma(i+j+m+m_l+k-0.5) \theta^k}{k! \alpha_3^{i+j+m+m_l+k-0.5}} + \sum_{k=0}^{i+m-1} \sum_{l=0}^{j+i+m-1} \frac{\Gamma(i+j+m+m_l+k+l-0.5) \theta^{k+l}}{k! l! \alpha_4^{i+j+m+m_l+k+l-0.5}} \right) \right], \quad (4)$$

respectively, where  $f_m$  is Doppler shift frequency,  $\rho$  is the

correlation coefficient,  $m$  and  $m_l$  are Nakagami parameters describing fading severity of desired signal and CCI, respectively, average SIR is  $S = \Omega_s / \Omega_l$  and

$$\delta = \frac{m \mu_{th}^2}{\Omega_s (1-\rho)}, \quad \beta = \frac{\sqrt{m_l \Omega_s + m \Omega_l \mu_{th}^2}}{\Omega_s^{m+k} \Omega_l^{m_l}}, \quad \gamma = \frac{m_l}{\Omega_l}, \quad \chi = m_l S, \quad \theta = m \mu_{th}^2 / (1-\rho), \quad \alpha = \rho^{i+j} / (i! j! \Gamma(m)), \quad \alpha_1 = \chi + \theta, \quad \alpha_2 = \chi + 2\theta, \quad \alpha_3 = \chi + (2+\rho)\theta, \quad \alpha_4 = \chi + (3+\rho)\theta, \quad \alpha_5 = \chi + (1+\rho)\theta.$$

The outage probability of the output SIR envelope,  $F_{\mu}(\mu_{th})$ , of the proposed dual and triple-branch SC diversity system can be obtained using [8], [9]

$$F_{\mu}(\mu_{th}) = 1 - \sum_{k=0}^{\infty} \frac{\rho^k (1-\rho)^m}{k! \Gamma(m)} \times \left[ 2\Gamma(m+k) - m^{m+k} \mu_{th}^{2m+2k} \sum_{p=0}^{m_l-1} \frac{m_l^p (1-\rho)^p \Gamma(m+k+p)}{p!} \times \left( \frac{\Omega_s^p \Omega_l^{m+k}}{(m \Omega_l \mu_{th}^2 + m_l \Omega_s (1-\rho))^{m+k+p}} + \frac{\Omega_s^p \Omega_l^{m+k}}{(m \Omega_l \mu_{th}^2 + m_l \Omega_s (1-\rho))^{m+k+p}} \right) - \sum_{k=0}^{\infty} \sum_{l=0}^{m+k-1} \frac{\rho^k (1-\rho)^m}{k! l! \Gamma(m)} \left[ \frac{\Gamma(m+k+l)}{2^{m+k+l-1}} - m^{m+k+l} \mu_{th}^{2m+2k+2l} \sum_{p=0}^{m_l-1} \frac{m_l^p (1-\rho)^p \Gamma(m+k+l+p)}{p!} \times \frac{2\Omega_s^p \Omega_l^{m+k+l}}{(2m \Omega_l \mu_{th}^2 + 2m_l (1-\rho))^{m+k+p+l}} \right] \right] \quad (5)$$

and

$$F_{\mu}(\mu_{th}) = 1 - \left\{ \sum_{i,j=0}^{\infty} \alpha \left[ \frac{2\Gamma(i+j+m)}{\Gamma(j+m)(1+\rho)^{i+j+m}} \times \left( (1-\rho)^m \left( (j+m-1)! - \sum_{k=0}^{i+m-1} \frac{(j+m+k-1)!}{2^{j+m+k} k!} \right) - \sum_{l=0}^{i+j+m-1} \frac{(1+\rho)^l}{l!} \left( \frac{(j+m+l-1)!}{(2+\rho)^{j+m+l}} - \sum_{k=0}^{i+m-1} \frac{(k+l+j+m-1)!}{k! (3+\rho)^{k+l+j+m}} \right) \right) - \sum_{p=0}^{m_l-1} \frac{\chi^p (m \mu_{th}^2)^{j+m}}{p! (1-\rho)^j} \left( \frac{(j+m+p-1)!}{\alpha_1^{j+p+m}} - \sum_{k=0}^{i+m-1} \frac{(j+m+p+k-1)! \theta^k}{k! \alpha_2^{j+m+p+k}} \right) \right] \right\}$$

$$\begin{aligned}
 & - \sum_{l=0}^{i+j+m-1} \frac{(1+\rho)^l \theta^l}{l!} \\
 & \times \left( \frac{(j+m+p+l-1)!}{\alpha_3^{j+m+p+l}} - \sum_{k=0}^{i+m-1} \frac{(p+k+l+j+m-1)! \theta^k}{k! \alpha_4^{p+k+l+j+m}} \right) \Bigg) \\
 & + (1-\rho)^m \left( \frac{(i+j+m-1)!}{(1+\rho)^{i+j+m}} - \sum_{l=0}^{j+m-1} \frac{(i+j+m+l-1)!}{l! (2+\rho)^{i+j+m+l}} \right. \\
 & \left. - \sum_{k=0}^{i+m-1} \frac{1}{k!} \left( \frac{(i+j+m+k-1)!}{(2+\rho)^{i+j+m+k}} - \sum_{l=0}^{j+m-1} \frac{(i+j+m+k+l-1)!}{l! (3+\rho)^{i+j+m+k+l}} \right) \right) \\
 & - \sum_{p=0}^{m_j-1} \frac{\chi^p \theta^{i+j} (m \mu_{th}^2)^m}{p!} \\
 & \times \left( \frac{(i+j+m+p-1)!}{\alpha_5^{i+j+m+p}} - \sum_{l=0}^{j+m-1} \frac{(i+j+m+l+p-1)! \theta^l}{l! \alpha_3^{i+j+m+l+p}} \right. \\
 & \left. - \sum_{k=0}^{i+m-1} \frac{\theta^k}{k!} \left( \frac{(i+j+m+k+p-1)!}{\alpha_3^{i+j+m+k+p}} \right. \right. \\
 & \left. \left. - \sum_{l=0}^{j+m-1} \frac{(i+j+m+k+l+p-1)! \theta^l}{l! \alpha_4^{i+j+m+k+l+p}} \right) \right) \Bigg) \Bigg\}.
 \end{aligned} \tag{6}$$

### III. SIMULATION RESULTS

The architecture of sum-of-sinusoids-based Nakagami- $m$  simulator is depicted in Fig. 1 [7].

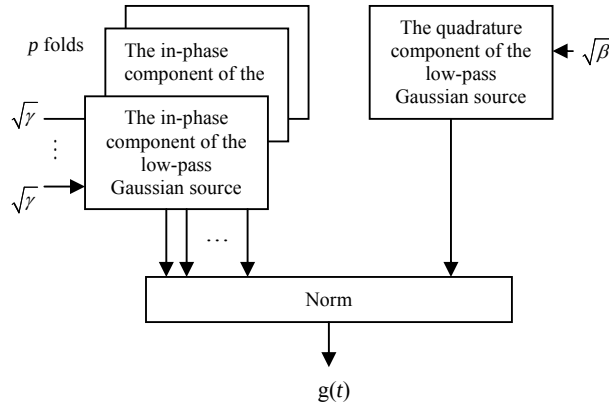


Fig. 1. The block diagram of sum-of-sinusoids-based Nakagami- $m$  simulator

The corresponding composite signal is

$$g(t) = \sqrt{\gamma \sum_{k=1}^p g_{I,k}^2(t) + \beta g_Q^2(t)}, \tag{7}$$

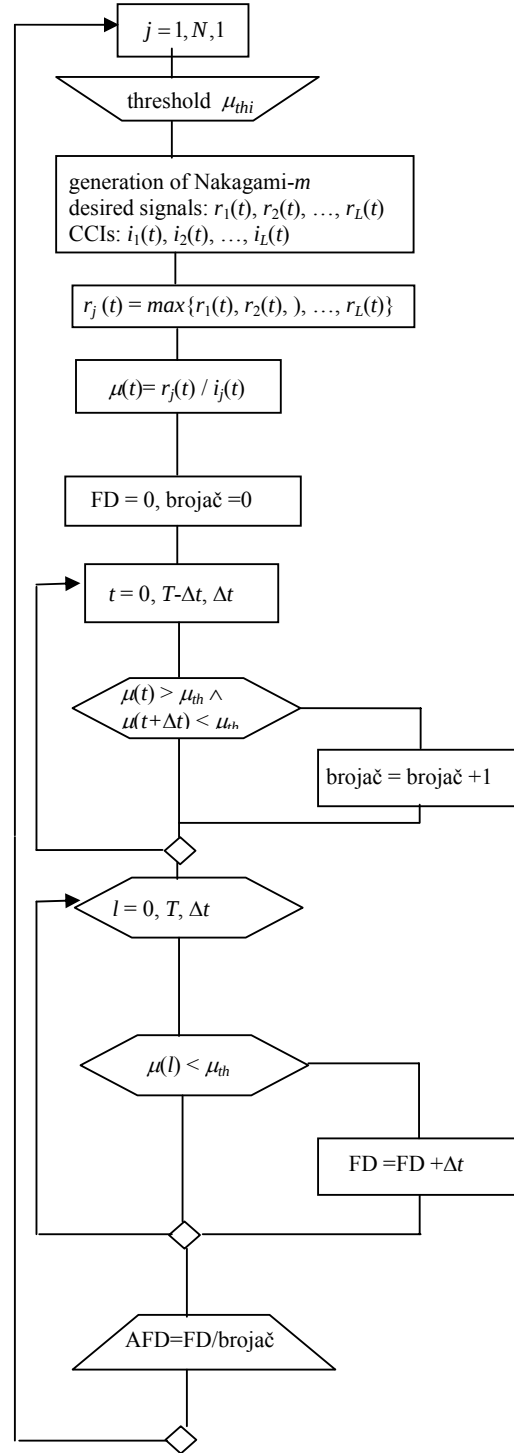


Fig. 2. The algorithm for simulation of AFD of considered  $L$ -branch SC receiver

where

$$g_I(t) = 2\sqrt{\frac{2}{N}} \times \left[ \sum_{n=1}^M \cos \Phi_n \cos(\omega_n t + \Psi_n) + \sqrt{2} \cos \Phi_n \cos(\omega_N t + \Psi_N) \right], \quad (8)$$

$$g_Q(t) = 2\sqrt{\frac{2}{N}} \times \left[ \sum_{n=1}^M \sin \Phi_n \cos(\omega_n t + \Psi_n) + \sqrt{2} \sin \Phi_n \cos(\omega_N t + \Psi_N) \right], \quad (9)$$

$$\gamma = \frac{2pm \pm \sqrt{2pm(1+p-2m)}}{p(1+p)} \quad (10)$$

and

$$\beta = 2m - \gamma p \quad (11)$$

with  $p = [2m]$ ,  $N = 4M+2$ ,  $\omega_n = 2\pi f_m \cos(2\pi n/N)$ ,  $\Phi_n = n\pi/M$ ,  $\Phi_N = 0$  and  $\psi_j$  is random phase uniformly distributed in the range  $(-\pi, \pi]$ .

Figure 2 describes AFD simulation process for desired signal based SC system operating in interference-limited Nakagami- $m$  environment.

Figures 3 and 4 show simulation and numerical results, evaluated using program packages MatLab and Mathematica, respectively, for uncorrelated ( $\rho \rightarrow 0$ ) dual and triple SC diversity system [9] in environments under different fading severity.

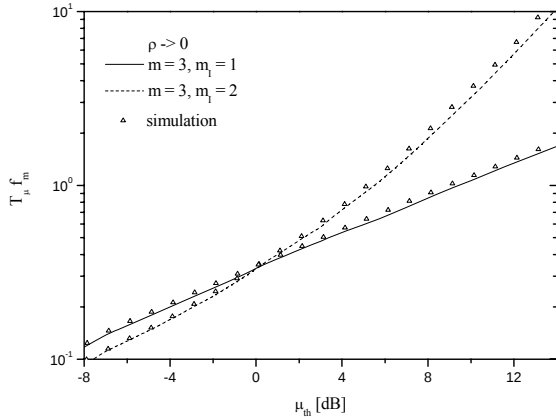


Fig. 3. AFD of dual SC diversity system

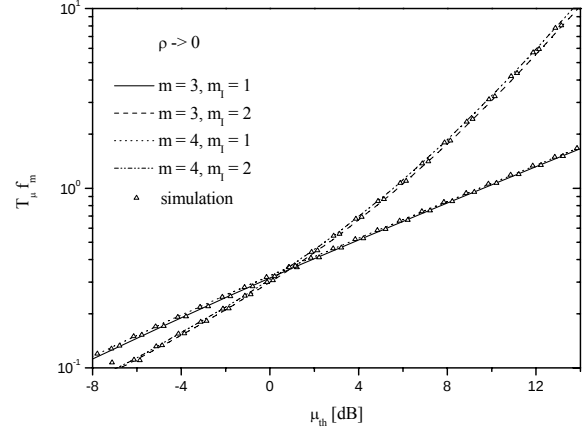


Fig. 4. AFD of triple SC diversity system

The great agreement between numerical and simulation results is evident regardless of number of diversity branches or fading severity.

Aiming to achieve greater precision, number of chosen oscillators is  $M = 500$ . In all simulations maximum Doppler frequency is  $f_m = 100$  Hz causing selected  $\Delta t = 10 \mu s$ .

#### IV. CONCLUSION

This work presents the extension of [12] and it is results of intention to verify previously published theoretical results. AFD as important dynamic performance characteristic is simulated for SC diversity system with two and three uncorrelated branches in Nakagami- $m$  fading environment in the presence of CCI. Simulation results obtained using program package Matlab show great agreement with earlier published numerical results calculated using program package Mathematica.

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